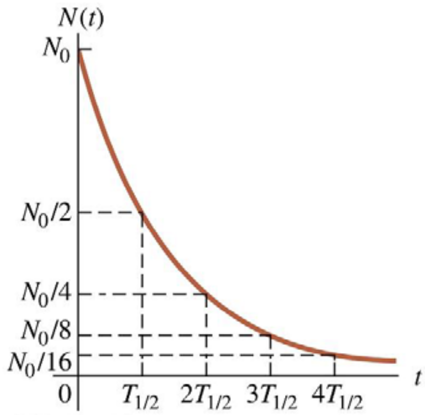

Natural Logarithms

A logarithm is an exponent. We are most familiar with the base 10 number system:

$10^0 = 1$, and log 1 = 0
$10^1 = 10$, and log 10 = 1
$10^2 = 100$, and log 100 = 2

However, many phenomena in nature follow an exponential decrease or increase. Natural or base *e* logs are based on *e* = 2.718281828423536…, which is just a number like π. You will use the **[ln]** and **[e^x]** keys on your calculator.

Calculate the following:

e^1 = ____________________

ln 465 = ________________

e^0 = ____________________

$e^{6.1420}$ = _________________

e^{-1} = ____________________

ln 2 = ____________________

ln 2.7183 = ______________

ln ($\frac{1}{2}$) = _________________

Sometimes we must solve an equation like $I = I_o e^{-\mu x}$ for x. This equation says that if you start with some quantity I_o, after a distance x the quantity will be I. You might also see something like $R = R_o e^{-\lambda t}$ which says that the quantity R_o is reduced to R after time t has passed. Both μ and λ are constants. Example: Solve $I = I_o e^{-\mu x}$ for x when I = $\frac{1}{2}$ I_o.

$$I = I_o e^{-\mu x} \Rightarrow \frac{1}{2} I_o = I_o e^{-\mu x}$$

$$\frac{1}{2} = e^{-\mu x}$$

$$\ln\left(\frac{1}{2}\right) = \ln\left(e^{-\mu x}\right) \Rightarrow \ln\left(\frac{1}{2}\right) = -\mu x$$

$$x = \frac{\ln\left(\frac{1}{2}\right)}{-\mu} = \frac{\ln 2}{\mu} = \frac{.6931}{\mu}$$

A numerical example: Suppose μ = .01/cm. Find x for I = ½ I_o. (This is referred to as the Half Value Length or Half Intensity Thickness.) We can use the results of the previous symbolic example.

$$x = \frac{.6931}{\mu}$$
$$x = \frac{.6931}{.001/\text{cm}}$$
$$x = 6.93\ \text{cm}$$

Now let's work a numerical answer where you use the **[e^x]** key. $R = R_o e^{-\lambda t}$ describes the activity of a radioactive substance. Suppose R_o = 1000 Curies, a common unit of activity. Suppose Iodine 131 (half life 8.04 days) is delivered to a thyroid patient via an IV. The decay constant, λ, is .0862/day. Find the activity R after two days have passed. Put the numbers in your calculator and make sure you can get the answer.

$$R = R_o e^{-\lambda t}$$
$$R = (1000\ \text{Curies})e^{-(.0862/\text{day})(2\ \text{days})}$$
$$R = (1000\ \text{Curies})e^{-(.1724)}$$
$$R = (1000\ \text{Curies})(.84164)$$
$$R = 842\ \text{Curies}$$

Challenge Problem: How long would you need to wait for the ^{131}I activity to drop to only 5% of what it was originally? Hint: Let R = .05R_o.